Dynamic Predictions from Time Series Data - An Artificial Neural Network Approach

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ABSTARCT

A hybrid approach, incorporating concepts of nonlinear dynamics in artificial neural networks (ANN), is proposed to model time series generated by complex dynamic systems. We introduce well known features used in the study of dynamic systems - time delay τ and embedding dimension d - for ANN modelling of time series. These features provide a theoretical basis for selecting the optimal size for the number of neurons in the input layer. The main outcome of the new approach for such problems is that to a large extent it defines the ANN architecture and leads to better predictions. We illustrate our method by considering computer generated periodic and chaotic time series. The ANN model developed gave excellent quality of fit for the training and test sets as well as for iterative dynamic predictions for future values of the two time series. Further, computer experiments were conducted by introducing Gaussian noise of various degrees in the two time series, to simulate real world effects. We find rather surprising results that upto a limit introduction of noise leads to a smaller network with good generalizing capability.

Keywords: Nonlinear dynamics, Chaos, Artificial neural network, Time Series Analysis.

1. INTRODUCTION

The study of nonlinear dynamics and chaotic systems has provided new insights into the understanding of many complex physical systems and has contributed significantly to the development of nonlinear time series analysis[1,2,3]. Formally, we expect that the complex dynamics is expressible in the form of a set of coupled nonlinear differential equations or discrete time maps involving all the system variables. The dynamics of the system, in such ideal cases, may then be investigated by solving the known equations. In most realistic situations, however, such detailed information about all the variables, their interactions and dynamics is not available. Instead, for modelling the system we have time series data of one or more significant system variables. The task of reconstructing the dynamics of the system from the time series data is therefore of primary importance. In this context, state space methods of recovering the dynamics of the system in terms of the known time series have been proposed[1,2,4]. The captured dynamics may then be used to estimate invariants of dynamics as well as to make short-term predictions. Much of the success of neural networks is due to such characteristics as parallel processing, nonlinear processing, and nonparametric as well as distributed representation[5]. The prototypical use of neural nets is in *structural pattern recognition*[5], where a collection of features - visual, semantic or otherwise - is presented to a network and the network must categorise the input feature pattern as belonging to one or more classes. In contrast, *temporal pattern recognition* involves processing of patterns that evolve over time. The appropriate response at a particular point in time depends not only on the current input, but potentially on all previous inputs. Several researchers, including Miller et al[7], Weigend, Hubernman and Rumelhart[6], Lapedes and Farber[8], Weigend and Gershenfeld[3] etc. have discussed the paradigms for system modelling using neural networks and time series data for the actual system. In this note, we, therefore, combine concepts and methods of nonlinear dynamics in the framework of ANN to construct a model for the dynamics underlying the time series. We also address the issue of how the time-varying input sequence should be represented. The nature and quantity of information that must be presented to the network is domain dependent.

In order to develop a model, it is necessary to subject the time series to rigorous pre-analysis phase. The pre-analysis characterizes the dynamic system and provides valuable information about its nature. In many cases, through characterization, it is possible to find out whether the system is i) linear or nonlinear, ii) deterministic or stochastic, iii) regular or chaotic. For this purpose various measures of the time series such as auto-correlation, power spectrum, average mutual information, number of false nearest neighbours, Lyapunov exponents and DVS (deterministic vs stochastic) plot are evaluated. Kulkarni et al[9,10] discuss a similar approach for analysing and modelling electroencephalograph (EEG) time series data for capturing the underlying dynamics of the system. It is important at this stage to understand the difference between deterministic chaotic and stochastic (random) systems. This is because the former can be modelled dynamically but the latter cannot be - one has to use probabilistic notions. A chaotic system is one which exhibits extreme sensitivity to initial conditions. As a result a small error at time $t = 0$ grows exponentially with time t. Thus, even for such completely deterministic systems the time evolution cannot be predicted beyond a certain point as errors become large. Following the pre-analysis of the time series data an appropriate ANN model is constructed if the system is dominantly deterministic and predictions are made and tested.

The organization of the paper is as follows: In section 2, we describe the basis on which a model for the time series has been developed. Section 3 outlines the implementation details of the model in the framework of ANN. Various parameters to characterize the system generating the time series are described in section 4. In section 5, we illustrate the methodology by applying it to a periodic time series defined by the sine function whereas in section 6 the method is used for a chaotic time series obtained by solving Lorenz equations. In each case we consider a time series of about 1000 data points. This is in contrast to other studies in the literature[3] where the time series has 10,000 points. The reason we choose a smaller set is because it is closer to real world situation. We also consider in sections 5 and 6 effects of adding noise to these computer generated time series since data of real system always has some noise. Finally, a summary and some conclusions are given in section 7.

2. MODELLING THE TIME SERIES

The modelling of complex dynamics from an observed time series is based on the concept of state space reconstruction as advanced by Packard et al[1] and Ruelle et al[2]. They proposed a method called the method of delays to reconstruct the phase space using delay coordinate vectors derived from a scalar time series $\{X(t) : t = 1, ...N\}$ measured using time interval Δt. A delay coordinate vector $\vec{X}_i$ is defined as

$$\vec{X}_i = \left\{X_i,\ X_{i-\tau}, ...X_{i-(d-1)\tau}\right\} \tag{1}$$

where τ is an appropriately chosen delay which is an integer multiple of sampling time interval Δt, and d is an integer embedding dimension. If the solution to the equations lies on an attractor of dimension d_A, then choosing an integer $d > 2d_A$ is a sufficient condition for unfolding the attractor from the scalar time series. The delay coordinate vectors or embedding vectors when evolved in time determine the essential geometric and topological structures of the attractor of the complex system in the multi-dimensional space of its variables. Since the components of delay coordinate vectors are involved in modelling the system, it is vital that these components be essentially independent. This has been generally ensured by choosing a proper value of delay τ. Another crucial parameter in this geometric reconstruction of phase space is the embedding dimension d. Essentially it is the dimension of (Euclidean) phase space in which the attractor is embedded or the minimum number of dynamic variables needed to model the system. In view of these considerations, the time evolution of the variable X can be now given in the form of a smooth map

$$X_{i+T} = f\left\{X_i,\ X_{i-\tau}, ...X_{i-(d-1)\tau}\right\} \tag{2}$$

where T is the prediction step.

3. MODEL IMPLEMENTATION AND FORECASTING

In order to implement the model in eq.(2), we need to know the crucial parameters time delay τ, the embedding dimension d and the general characteristics of the time series. To be specific, for a deterministic system, the characteristics of the series may indicate if the functional form of the model in eq. (2) should be linear or nonlinear. Since the exact functional form in equation (2) is not known, a non-parametric model in the framework of ANN[3] would in general be very useful. Consequently, we employ the widely used multi-layer feed-forward network architecture with backpropagation scheme to model the time series. The number of neurons in input and output layers are $N1 = d$ (embedding dimension) and $N2 = 1$ respectively. The number of neurons in the hidden layer was varied to obtain an optimum network with least amount of root mean square (rms) error. The transfer function used was hyperbolic tangent for the hidden layer and linear one for the output layer. The network is trained using the training set consisting of embedding vectors and corresponding scalar prediction components (see eq. 2) derived from the series. The last held-back values of the series, not included in the training set, were used to compare the corresponding predicted values of the model.

The future values were forecast using the iterated single step predictions. The direct multistep predictions need different networks to be trained for each value of T and are thus very compute intensive. They had also been reported[11] to be not as good as iterated single step predictions. In iterated single step prediction, the previously predicted values were used to evaluate future values. Thus the errors in the earlier predicted values get propagated to the subsequently predicted values. The strategy thus seems to be a very sensitive test of how accurately the dynamics has been captured. The measure for the quality of prediction is given by the normalized mean square error (NMSE) defined as

$$\text{NMSE} = \frac{\dfrac{1}{N}\displaystyle\sum_{i=1}^{N}(\text{error})_i^2}{\text{variance of } N \text{ (data points)}}$$

where error(i) is the difference between i-th observed and predicted values. The measure takes into account both the number of terms and range of two sets to be compared. A value of $NMSE = 1$ corresponds to predicting the unconditional mean.

4. PRE-ANALYSIS OF TIME SERIES

In order to build an appropriate model, it is necessary to investigate the characteristics of the series. As stated earlier, the characterization gives both qualitative and quantitative information about the dynamical system that generates the series. We discuss here some of the useful quantities and their interpretation to understand the nature of the series. Since some of these parameters have to be computed in the reconstructed phase space, corresponding algorithms require a relatively large data set and are also sensitive to the noise in the data which may lead to less accurate estimations of crucial parameters such as time delay τ and embedding dimension d. We return to some of these points in Secs. 5 and 6 when we add noise to the computer generated series.

4.1 Auto-correlation function

A time series $\{X(t) : t = 1, 2, ..., N\}$ is usually generated by measuring a dynamical scalar variable at some constant time interval Δt. The behaviour of an auto-correlation function[4,12] (ACF) indicates how the successive values are correlated with each other. An auto-correlation value $a(L)$ for lag L is defined as

$$a(L) = \frac{\sum_{i=1}^{N} \left(X(i+L) - \bar{X}\right)\left(X(i) - \bar{X}\right)}{\sum_{i=1}^{N} \left(X(i) - \bar{X}\right)^2} \tag{4}$$

where $\bar{X}$ is the mean of the series. The ACF is a plot of $a(L)$ versus L for various values of lag L. If ACF falls to zero slowly, it implies that the series is highly correlated and may have a dominant deterministic component in the dynamics. However, if ACF falls to zero quickly, it may be that the series is either stochastic or chaotic. In general the periodicity of the series is well reflected in the periodicity of ACF.

4.2 Average mutual information

A quantity called average mutual information[4] (AMI) is a probabilistic measure which is a generalization of auto-correlation function to nonlinear domain. It is defined by the expression

$$I(L) = \sum_{i=1}^{N} P\left(X(i), X(i+L)\right) \log_2 \left[\frac{P\left(X(i), X(i+L)\right)}{P\left(X(i)\right) P\left(X(i+L)\right)}\right] \tag{5}$$

where $P(X(k))$ is probability of measuring $X(k)$ and $P(X(k), X(k+L))$ is the joint probability of measuring $X(k)$ and $X(k+L)$. The time delay τ used in eq.(1) is generally chosen to be the value of L at which the first minimum in the average mutual information (AMI) plot of $I(L)$ versus L appears. Note that if there is no clear minimum in the plot, one takes τ to be that value of lag L at which $I(L) = I_{max}/5$. The time delay τ can also be determined to be the lag at which either the ACF crosses the zero line or it drops to 1/e. However, the average mutual information method has an advantage that it considers all kinds of relations, not only the linear ones as in ACF. The best value of τ is perhaps the one at which the phase space plot shows maximum scatter.

4.3 Power spectrum

The power spectrum is the fourier transform of the auto-correlation function. The power spectrum is the plot of power versus frequency and clearly brings out the distinct periodicities in the form of sharp power peaks. However a broad band spectrum may mean that the series is either stochastic or chaotic. In order to differentiate it further, one needs to examine the fall of the power spectrum. An exponential fall[13] with respect to frequency refers to chaotic series while a power-law fall indicates that the spectrum is stochastic. It may be noted that the sampling frequency and the length of the series need to be determined on the basis of periodicities to be included in the model. The power spectrum may also show the presence of stochastic or coloured noise in the form of low power in the background.

4.4 Embedding dimension

As stated earlier, the embedding dimension determines the minimum number of dynamic variables needed to model the system. Several methods have been proposed to determine the embedding dimension d from the time series data. The method[4] of false nearest neighbours is geometric in construct. It rests on the fact that points in d dimension space may become neighbours if projected into a lower dimension. Thus one calculates the number of false nearest neighbours as a function of a variable dimension D. The value of D at which the number of false nearest neighbours goes to zero is taken to be the embedding dimension. This method is intuitively appealing and computationally quite robust. The plot of the number of false nearest neighbours (FNN) as a function of dimension D is very revealing. If the curve reaches zero and remains zero for higher dimension, it implies that the dynamics is deterministic. However if the curve starts rising for higher dimension, it may imply that deterministic dynamics is to some extent diluted by stochastic or coloured noise. If it never falls to zero, the dynamics may be interpreted to be predominantly stochastic. It may be stressed that the data points in the series should not only be sufficient in number but they should also traverse the major part of the attractor. The goodness of time series prediction largely depends on the fact that dynamics is dominantly deterministic.

4.5 Lyapunov exponents

Chaos in deterministic system implies a sensitive dependence on initial condition. Based on this property, Lyapunov exponents provide a diagnostic tool to determine whether or not a system is chaotic. The basic idea is that for chaotic systems if two trajectories start close to one another in phase space, they will move exponentially away from each other for small times on the average. Thus if d_0 is a measure of the initial distance between the two starting points, at a small but later time the distance is

$$d_t = d_0 \exp\left(\lambda t\right) \tag{6}$$

where λ is called a Lyapunov exponent. Lyapunov exponent is defined for each dimension of phase space and thus what we get is Lyapunov spectrum. There are many algorithms to determine the values of exponents from the time series. We have used algorithm by Zeng et al[14] which can be used for short time series to evaluate the Lyapunov exponent spectrum. If the largest Lyapunov exponent is greater than zero, then the system is chaotic. Otherwise it is considered as regular motion.

4.6 DVS Plot

The DVS plot provides very broad qualitative understanding of dynamics of time series. The DVS algorithm[15] attempts to fit an auto-regressive linear model of the form

$$X_{(i+T)} = f\left(\vec{X}_i\right) \tag{7}$$

where f is a linear function and $\vec{X}_i$ is the i-th embedding vector as defined in section 2. In order to obtain the DVS plot, we fit a family of linear models varying the size of neighbourhood in a reconstructed phase space. The short-term predictive accuracy of the models is estimated as out-of-sample error on a held-back portion of the time series. We then plot the average out-of-sample errors as a function of the size of the neighbourhood to obtain what is called a DVS plot. If the underlying dynamics is indeed chaotic and low dimensional, the models with small neighbourhood should give more accurate forecasts than those with large neighbourhood. This is because the nonlinearity in small neighbourhoods can be very well approximated by linear models. However if the underlying dynamics is stochastic or chaotic with high dimension, then very local models (in small neighbourhoods) will give less accurate forecasts as they tend to fit the noise in addition to the signal compared to global models with large neighbourhood. One can also vary the embedding dimension systematically to find which value of embedding dimension gives smallest error. This may be an alternative way of determining the value of embedding dimension. We have obtained DVS plots using local linear approximation as described here. It may be possible to obtain the DVS plots using nonlinear models using radial basis functions or neural networks.

5. MODELLING A PERIODIC SYSTEM

As a simple illustration we first model the dynamics of a sine series, using sampled data which provided the time series. The system output, y was given as follows:

$$y = A * \sin(2\pi f + \theta)$$

where A = 1.0, f = 2.0 and θ = 0 were the chosen as parameter values.

The sampling frequency for generating the time series data was chosen to be 256 Hz and thus the time step ΔT was taken as 1/256. The network was trained with the data containing about 7 full cycles which provided 950 data points, while the next 300 data points had to be predicted during the testing phase. The time delay (τ) was determined from the evaluation of average mutual information and found to have the value 4. Based on the value of τ = 4 , the embedding dimension, d, value was found to be 3.

A multilayer perceptron with $3 \times 7 \times 1$ configuration was trained using the input-output vectors obtained with the above parameters. The network converged to the NMSE value of 0.00009 and the fit of the trained network to the observed 950 data values was nearly perfect. Using the trained network single step predictions were made for the next 300 values in the time series and again the fit was nearly perfect. However in order to determine whether the neural network has really extracted the underlying dynamics of the system (i.e. frequency, amplitude, phase etc. in case of this system) it is essential to test its performance for iterated single step predictions. Fig. 1 shows the iterated single step prediction values which demonstrate a complete failure in terms of all the system characteristics (i.e. frequency, amplitude etc.). The NMSE for the predicted values for the first 50 points is 208, which is extremely high and implies that the network is not able to generalize.

As a result, the number of neurons in the hidden layers were increased and the network was trained for various configurations. It was determined that a configuration of 3x20x1 was required for training the network for convergence to a low NMSE of 0.000003. Fig. 2 shows the observed values and the iterated single step predictions using this trained network for the next 300 points in the sine series. It is clearly evident that the network model has captured the characteristics of the complex system very well in terms of the frequency, amplitude, phase etc. In this case for the predicted 300 points the NMSE was 0.003.

For time series predictions often recurrent networks perform better than the traditional multilayer feed forward architecture. The network was then trained using the Backpropagation Through Time (BTT) method[16] using a correction for various step sizes. It was seen that a much smaller network architecture (3x6x1) was required in order to model the system in this case. See Fig. 3 for the plots of observed data versus iterated single step prediction values for the next 300 data points generated at the same sampling rate. It can be seen that the process of feedback adds the necessary non-linearity, therefore a smaller network is able to perform the same task.

At this stage, a new data set was generated by superimposing noise with gaussian distribution on the original sine series data set. Gaussian noise with mean = 0 and the standard deviation equal to some percentage of the standard deviation of the signal into which it is to be superimposed was generated. More precisely, the new series is[17,18]

$$Y_i = X_i + \alpha Z_i \;\; (i = 1, ...N)$$

where $\{X_i\}$ is the original series and the set $\{Z_i\}$ denotes N randomly chosen Gaussian variates with mean $\bar{Z} = 0$ and variance $\sigma_Z^2 = 1.0$. The parameters $\alpha \equiv k\frac{\sigma_X}{\sigma_Z} = k\sigma_X$ where σ_X^2 is the variance of the time series $\{X_i\}$. Further, k in per cent denotes the amount of noise we add to the original pure (noise free) time series $\{X_i\}$.

The signal to noise ratio (SNR) was varied such that the degree of noise k was initially 5% and then it was increased in steps of 5% to a total of 70%. The results of the fit and predictions for 5, 15 and 28% noise were excellent. Here we report the results for 28% noise. For the noisy data set the time delay (τ) as well as the embedding dimension (d) increased compared with the pure sine series data. Fig. 4 shows the plot of time delay and the embedding dimension. It was observed that the dip in the value for τ was not as prominent as seen in case of the pure data. In case of the d values the number of false neighbours remains zero throughout, indicating the dominantly deterministic nature of the system in the presence of noise upto 40%. As the noise increased beyond 40% the plot of number of false nearest neighbour versus variable dimension shows a different behaviour. Initially the number of false nearest neighbours decreases with D and then increases again. This is shown in Fig. 5. Such behaviour is indicative of stochastic nature of the system. As mentioned earlier statistical (not ANN) techniques are required to model the system in these cases. Thus one can expect the neural network to extract the trend and model the system for noise upto 40% since the signals are not random even though there is noise.

While modelling the time series with noise it was observed that for noise upto 15% the network had fewer parameters compared to the pure case. By the time the noise was 28% the network size had again become large. This is shown in Table 1. Based on the plots of Fig. 4, we obtained $\tau = 8$ and $d = 5$ for 28% noise. A network with at least 10 hidden neurons were required for convergence at a lower NMSE value of 0.05. Increasing the number of hidden layer neurons beyond this resulted in overfitting, such that the network converged to a lower NMSE value but the characteristic features like the frequency, amplitude etc. were not correctly extracted. Fig. 6 shows the plot of noisy data set with 28% noise and the fit obtained by training the neural network.

Figures 7a and 7b show the next 300 actual data points for this series along with the iterated single step prediction values. In this case the first input vector used for the iterated single step predictions was the noisy data vector. The NMSE of the predicted values with respect to noisy data was found to be 0.066. We observe that the plot for the iterated single step predictions contains jitters which subside as you predict further ahead in time. Figure 7a indicates the noisy input data values along with predicted values. It can be seen that the trend is extracted quite nicely in the presence of noisy data. Next we have compared these predicted values with the noise-free sine series values. The corresponding NMSE was 0.024 which is smaller than the value of 0.066 obtained with noisy data. This implies that the network has really captured the dynamics and has good generalizing capability. This was further reinforced when the iterated single step predictions were obtained using the noise-free sine series values as initial input values. The NMSE of these predictions with respect to the corresponding values of noise-free sine series was found to be 0.0036. Since the starting points have no error, the network could generate the original noise-free sine series to a great deal of accuracy. Consequently the plot in fig. 7b which shows the actual pure sine series values along with the predicted values is very smooth. It can be seen that the neural network models the system perfectly even after being trained using noisy data and the predictions are even better than the network trained with pure sine data (Fig. 2), even though the NMSE value at convergence is higher (0.05 vs. 0.000003) in this case. It may also be noted that the number of parameters in this network is smaller (71) than those in the network trained with pure sine seris (101) as seen in Table 1.

6. MODELLING A CHAOTIC TIME SERIES

The systematic methodology of time series prediction is next applied to a computer generated time series of variable X obtained by solving Lorenz equations given below.

$$\begin{aligned}\dot{X} &= \sigma(Y-X)\\ \dot{Y} &= \rho X - Y - XZ\\ \dot{Z} &= -\beta Z + XY\end{aligned}$$

The parameters used in the solution of these equations are $\sigma = 16.0$, $\rho = 45.92$ and $\beta = 4.0$. The first 5000 points in the solution have been omitted to get rid of transient effects. Subsequently the series of X variable has been calculated by integrating the equations using the time step of 0.001 second for better accuracy. However the actual time series used for present analysis has obtained using the sampling interval $\Delta t = 0.05$ by selecting every 50-th value from the calculated series. Thus we obtained a Lorenz's time series of 1050 values. Although we could have taken much longer time series with obvious associated advantages, we preferred the size of 1050 values which is a typical size generally encounterd in scientific time series analysis. It may also be noted that the Lorenz series is a well known example of chaotic series.

In the pre-analysis phase we obtained ACF plot (fig. 8a), AMI plot (fig. 8b), FNN plot (fig. 9), power spectrum (fig. 10) and DVS plot (fig. 11). We also calculated the largest Lyapunov exponent. The ACF plot showed that the values in the series are quite correlated and auto-correlation value goes to zero only at $lag = 25$. If we had to determine the time delay from ACF, we would obtain $\tau = 4$ where the ACF approximately assumes the value of 1/e. However, as discussed before, we preferred to deletrmine the value of time delay τ based on AMI plot. Thus we obtained the value of time delay τ to be 2, where the AMI plot (see fig. 8b) had a distinct minimum.

The value τ is the input to obtain FNN plot which enables us to get the value of embedding dimension. Using $\tau = 2$ we obtained the FNN plot (Fig. 9). It was clear from the plot that the value of embedding dimension for the present series is 3 where the plot reached its zero value. It is gratifying to know that the Lorenz series of finite size could yield the correct value of the dimension of the state space. Since in linear time series analysis τ is generally assumed to be always one, we also obtained the value of embedding dimension using $\tau = 1$. The fact that it happened to be 3 seems to be just coincidence. In principal it can be different.

Being a chaotic series, the power spectrum of the present series was worth examining. We obtained (fig. 10) the plot of log(power) versus frequency. It was satisfying to find that the plot was just a straight line confirming that power spectrum of chaotic signal has an exponential power fall. The conclusion that the present series is chaotic was further supported by the evaluation of the largest Lyapunov exponent which was found to be 1.48.

The further probe of the series by using DVS plot was quite useful. The DVS plot (fig. 11) was obtained using the embedding dimension $d = 3$. We found that for $d = 3$, the RMS error was minimum. The curve also showed that RMS error went on increasing as we enlarged the neighbourhood. These two observations led us to broad qualitative understanding that the present series belonged to low-dimensional, nonlinear and deterministic system.

Next we tried to model the series using the information obtained from pre-analysis We developed following two models, using eq. (2).

1. Using $\tau = 1$ and $d = 3$
2. Using $\tau = 2$ and $d = 3$

The models were implemented using multi-layer feed-forward ANN with a single hidden layer. The first 950 values of the series were used to train the network and 100 additional values were predicted using the trained network. The predictions were made using both single-step prediction (SSP) and iterated single-step prediction (ISSP) scheme as described earlier. The parameters and results of the models are given in Table 2. Figure 12 shows the plot of 50 predicted values of the series obtained using model 1. Figure 13 shows the plot of 50 predicted values of the sine series obtained using model 2.

It can be seen from table 2 that in both the models the SSP values are in excellent agreement with the observed values. So far as the ISSP values are concerned, the first model could not capture the dynamics properly despite its good quality fit to training set and SSP values. Thus a good fit and equally good SSP values may not gaurantee good ISSP values. In fact we found that the NMSE value for the first 30 values in the first model is 0.085 which is substantially larger than the corresponding value (0.0108) in the second model. In contrast the second model could give the ISSP values which are in good agreement with the corresponding observed 50 values. This implies that the considerations of nonlinear dynamics in modelling the series are quite crucial. We also found that the predictability of the model varied with the initial point chosen - i.e. whether it was 949, 950 or 951. Further, in all the cases the breakdown of the model occurred suddenly - as seen in Fig. 13. These features are due to the chaotic nature of the system as they indicate sensitivity to initial conditions.

We next introduce noise in the Lorenz series. It has been found that, unlike the regular sine series, the modelling and prediction results in the case of chaotic Lorenz series are very sensitive to noise. While the sine series could be successfully modelled upto k=40% of noise, the Lorenz series could tolerate noise only upto k=0.4%. Figure 14 shows the effect of increasing the noise from k= 0.4% to k = 1.6% on embedding dimension. Note that for the noise level $>$ k= 0.4%, the FNN plot shows a rise with increase in the varying dimension D. Even for very small noise level of k = 0.4%, the embedding dimension value of Lorenz's series changed to 4 from 3. This implies that the system becomes predominantly stochastic even at low level of noise. As before for lower values of noise we do observe a reduction in the number of parameters required to model the system as we go from pure (noise free) limit to noise of k = 0.4%. For larger values of k, however, we cannot model the system using ANN as it is predominantly stochastic as indicated by the FNN plot (Fig. 14).

For gaussian noise with $k = 0.2\%$ superimposed on the pure data set (1000 points) generated earlier, the τ and d values were calculated as $\tau = 2$ and $d = 3$. A neural network with configuration $3 \times 7 \times 1$ using the noisy data set has converged to an NMSE value of 0.002. Figure 15 shows the plot of actual data and the iterated single step prediction values by the trained network for the next 50 points sampled at the same rate. Again it was found that the fit was almost as good as in the case of the network trained with the pure data with $NMSE = 0.095$ for the first 50 predicted values.

7. SUMMARY AND CONCLUSIONS

In this paper, we have modelled computer generated periodic and chaotic time series using ANN techniques. Before modelling the system we studied the characteristics of the underlying dynamics of the time series using the tools of nonlinear dynamics. The analysis of time series data enabled us to partly determine the architecture of the network. More precisely, we argue that the number of neurons in the input layer ought to be equal to the embedding dimension d. Our results for iterated dynamic predictions establish the importance of the parameter τ (time lag). We have also examined the effect of adding white noise to the two model time series. It is explicitly demonstrated that upto a limit noise helps in the sense that the network is smaller and the predictions better and robust. However, as the noise level increases, the determinstic component gets diluted and modelling becomes difficult. This can be very well seen by the study of FNN plot in which the false nearest neighbours increases considerably for higher dimension. Our study has also revealed that the noise tolerance level is much higher (about 40%) in the regular sine series than in the chaotic Lorenz series (about 0.4%). Finally, in our view, this study establishes for the first time, a systematic way of constructing non-parametric models for time series data.

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Table 1

Network Configuration for Sine Series Prediction

Data	Configuration	No. of Parameters
Pure Sine Series	3-20-1	101
Sine Series + 5% Noise	4-4-1	25
Sine Series + 15% Noise	3-6-1	26
Sine Series + 28% Noise	5-10-1	71

Table 2

Configuration and Errors in Two Models of Lorenz Series

Model	Network configuration	NMSE (Training)	NMSE (SSP-50)	NMSE (ISSP-30)	NMSE (ISSP-50)	NMSE (ISSP-100)
1	(3-7-1)	0.0002	0.00008	0.0850	1.4580	1.8640
2	(3-7-1)	0.0003	0.00019	0.0108	0.0476	0.1570

Figure Captions

Fig. 1 Actual (dashed line) and iterated single step predictions (solid line) of the sine series using a multi-layer perceptron with configuration (3x7x1).

Fig. 2 Actual (dashed line) and iterated single step predictions (solid line) of next 300 (out-of-sample) values for the sine series. The network configuration was (3x20x1).

Fig. 3 Actual (dashed line) and iterated single step predictions (solid line) of next 300 values of the sine series using recurrent network with a configuration (3x6x1).

Fig. 4 a) A plot of average mutual information versus lag for the sine series with noise for SNR at k = 28%.
b) A plot of false nearest neighbours versus dimension for the data in (4a).

Fig. 5 A plot of false nearest neighbours versus dimension for the sine series with noise for SNR at k=56% (dashed line) and k=70% (solid line).

Fig. 6 Actual (dashed line) and fitted values (solid line) of the sine series with noise for SNR at k = 28%.

Fig. 7 The iterated single step prediction (solid line) values for the next 300 values of the sine series with noise for SNR at k = 28% using the network configuration (5x10x1) along with

a) The actual values (dashed line) of the data with noise and noisy data as a starting point for the predictions.

b) The actual values (dashed line) of the pure sine series and pure data as a starting point for the predictions.

Fig. 8 a) The auto-correlation function for the pure Lorenz series.
b) The plot of average mutual information versus lag for the pure Lorenz series.

Fig. 9 The plot of false nearest neighbours versus dimension with $\tau = 2$ for the pure Lorenz series.

Fig. 10 The plot of power spectrum for the pure Lorenz series.

Fig. 11 The DVS plot for the pure Lorenz series, with $\tau = 2$ and $d = 3$.

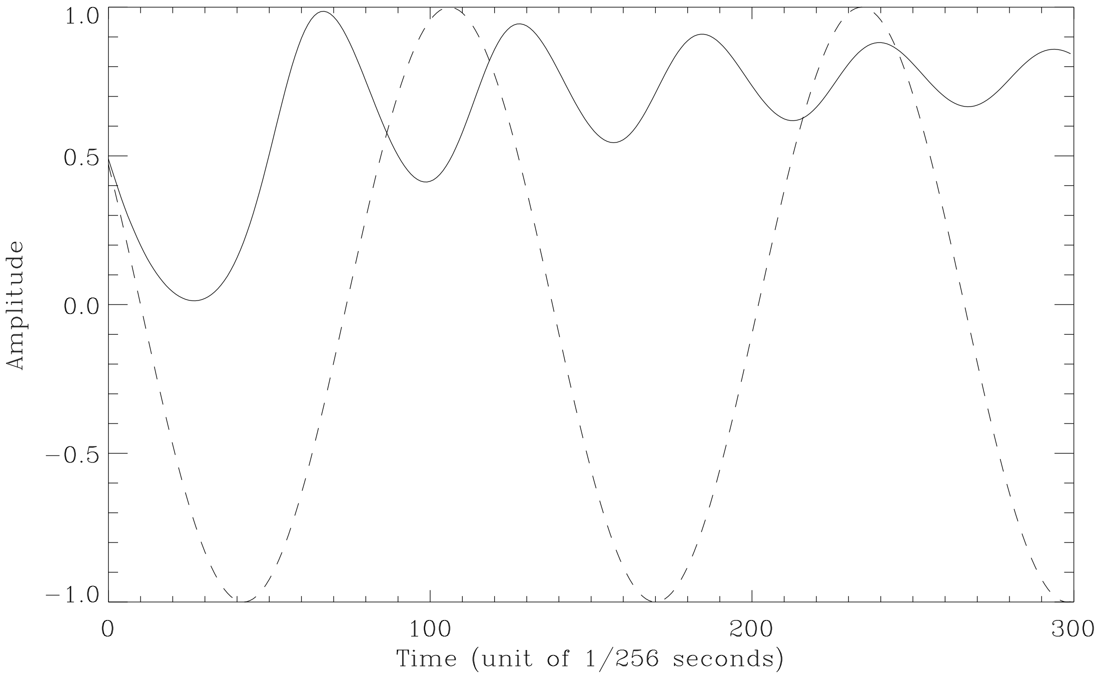
Fig. 12 The actual (dashed line), and iterated single step prediction (solid line) values fo the next 50 (out-of-sample) values of the Lorenz series using model 1.

Fig. 13 The actual (dashed line), and iterated single step prediction (solid line) values of the next 50 (out-of-sample) values of the Lorenz series using model 2.

Fig. 14 The plot of false nearest neighbours versus dimension for the noisy Lorenz series with
a) SNR at k = 0.4% (dashed line) b) SNR at k = 0.8% (dotted line) c) SNR at k = 1.6% (solid line)

Fig. 15 The actual (dashed line), and iterated single step prediction (solid line) values of the next 50 (out-of-sample) values of the noisy Lorenz series with SNR ar k = 0.2%. The network configuration was (3x7x1).

Figure 1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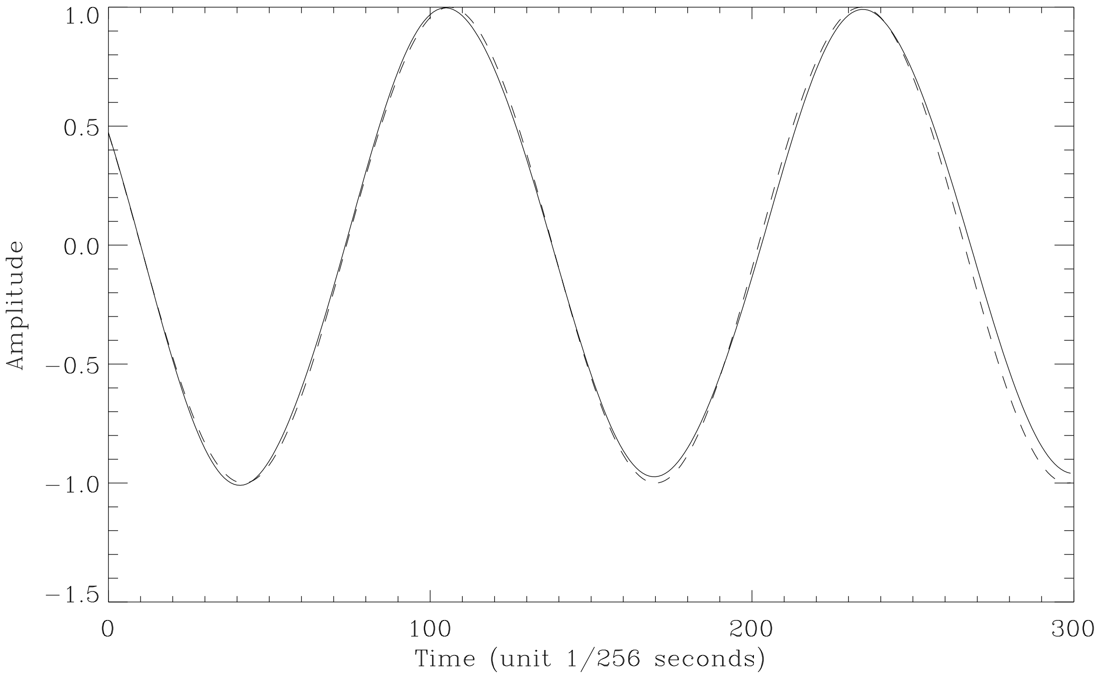
0.5
0.0
−0.5
−1.0
Amplitude
0
100
200
300
Time (unit of 1/256 seconds)

Figure 2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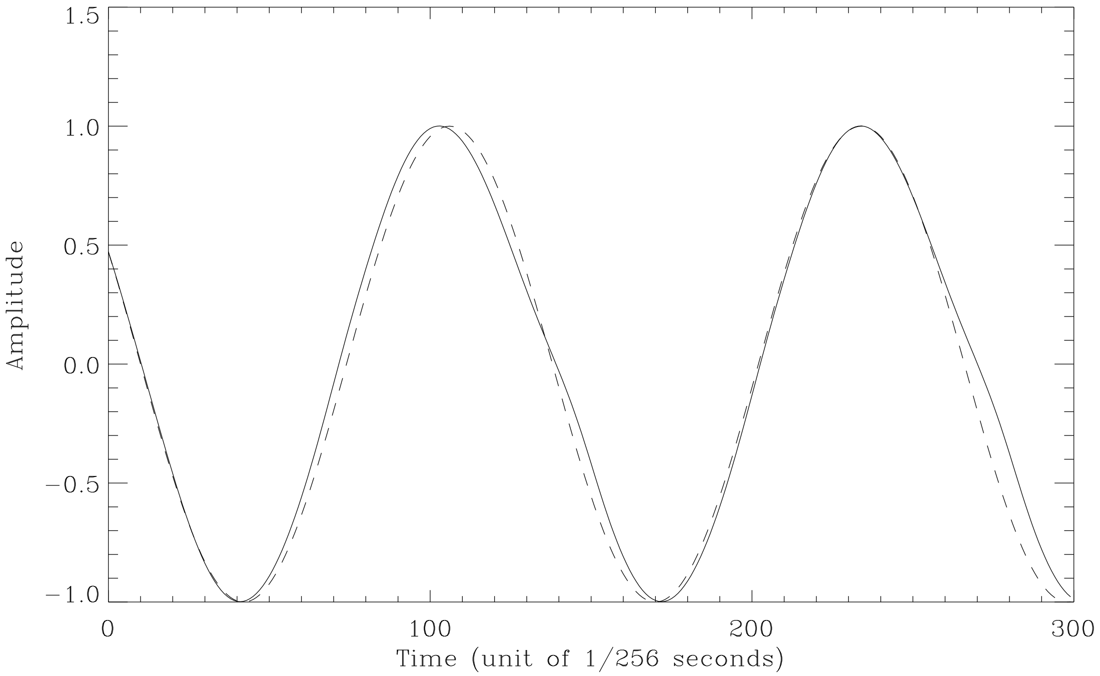
−0.5
−1.0
−1.5
Amplitude
0
100
200
300
Time (unit 1/256 seconds)

Figure 3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−0.5
−1.0
Amplitude
0
100
200
300
Time (unit of 1/256 seconds)

Figure 4(a)

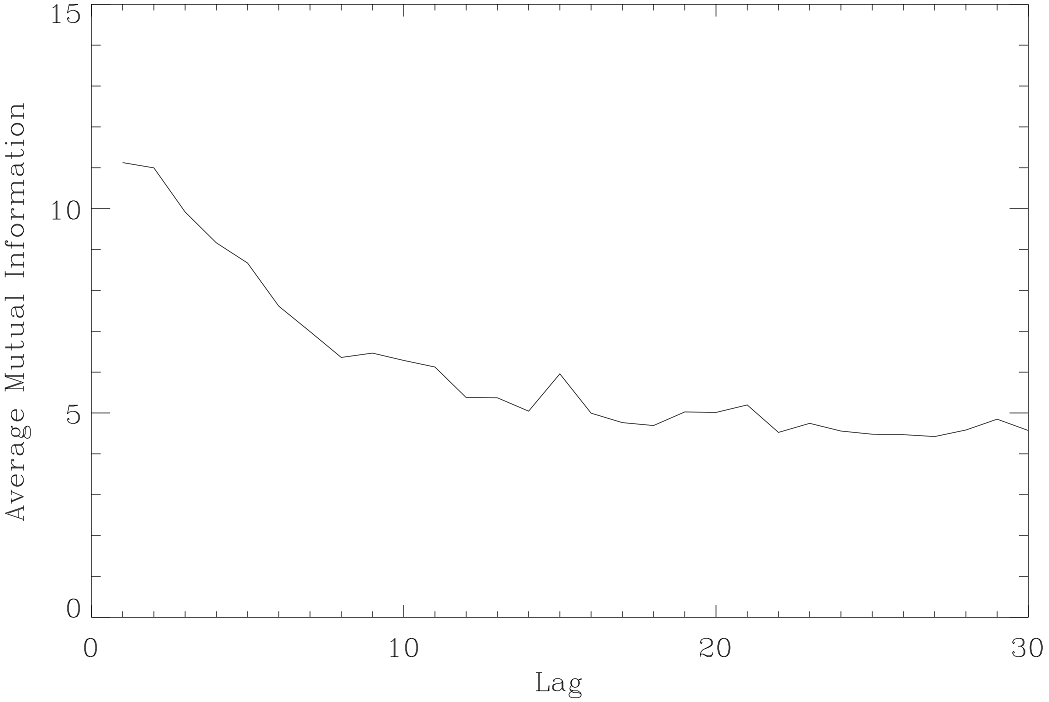
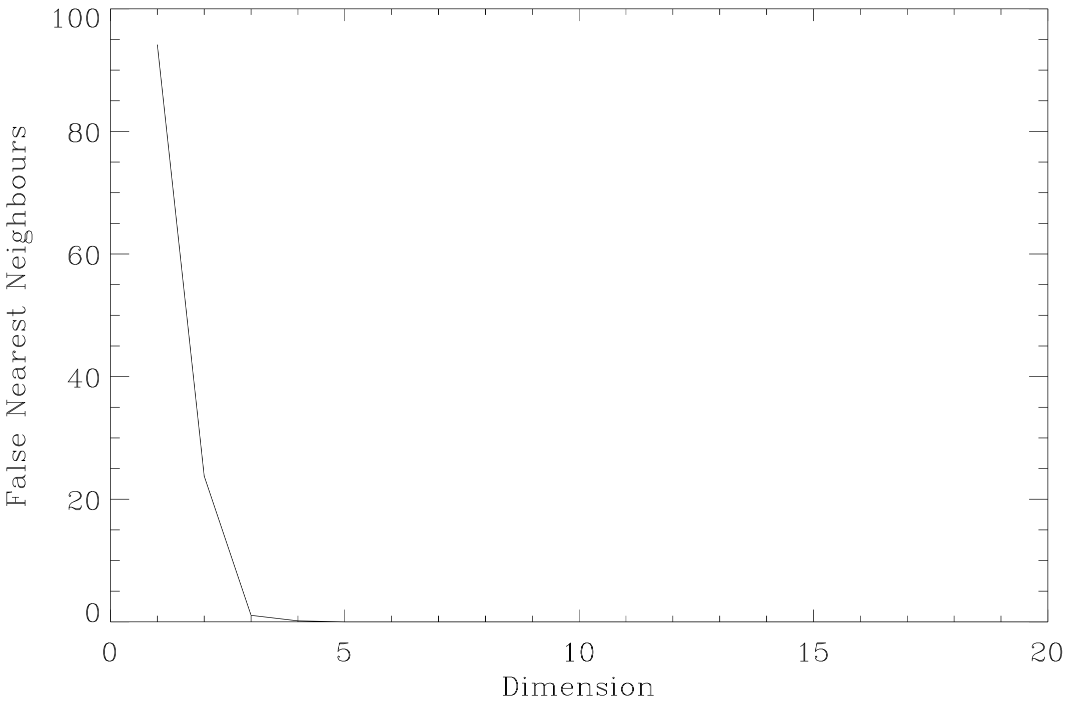


Figure 4(b)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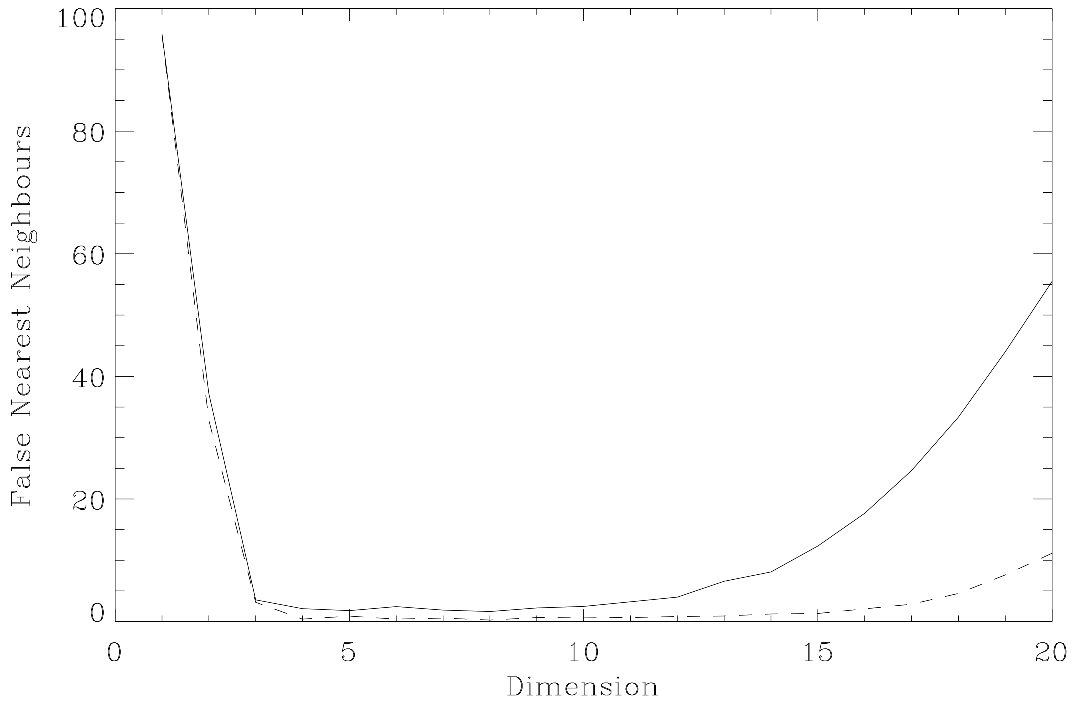
80
60
40
20
0
False Nearest Neighbours
0
5
10
15
20
Dimension

Figure 5


False Nearest Neighbours
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80
60
40
20
0
0
5
10
15
20
Dimension

Figure 6

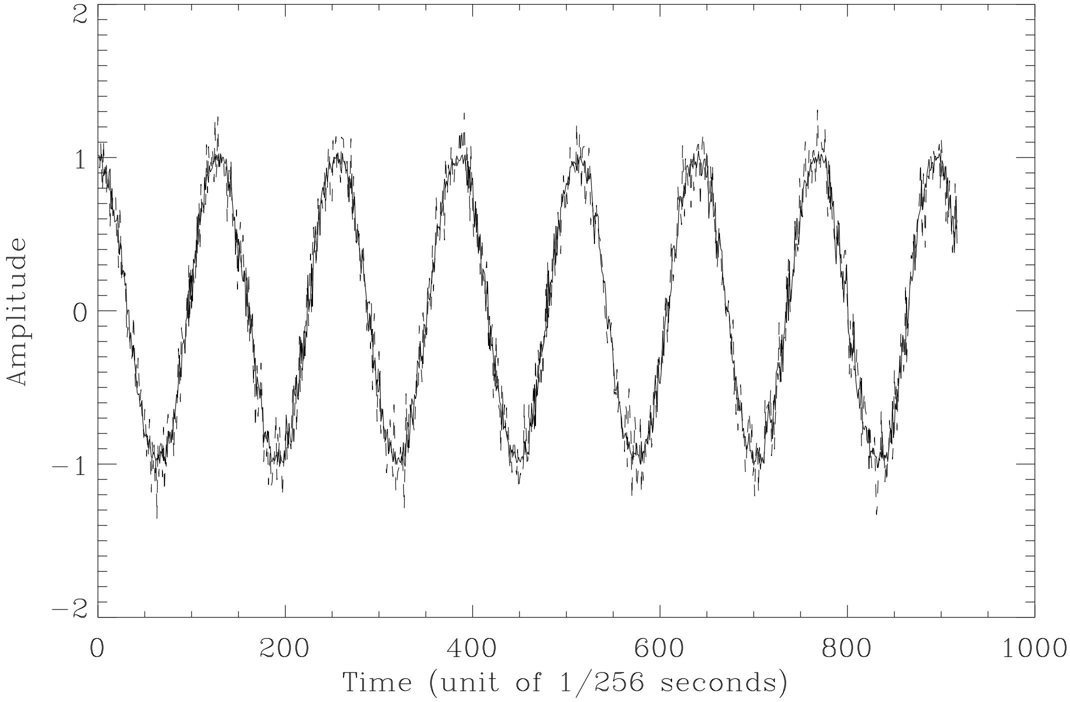
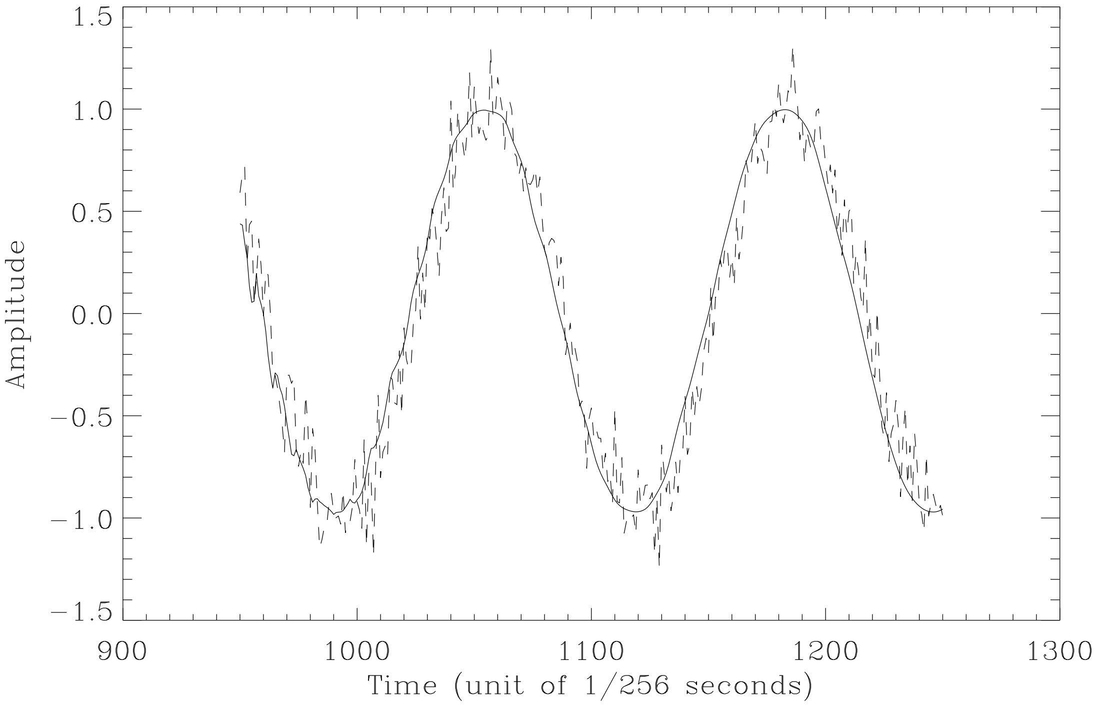

Amplitude
2
1
0
−1
−2
0
200
400
600
800
1000
Time (unit of 1/256 seconds)

Figure 7(a)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1.0
0.5
0.0
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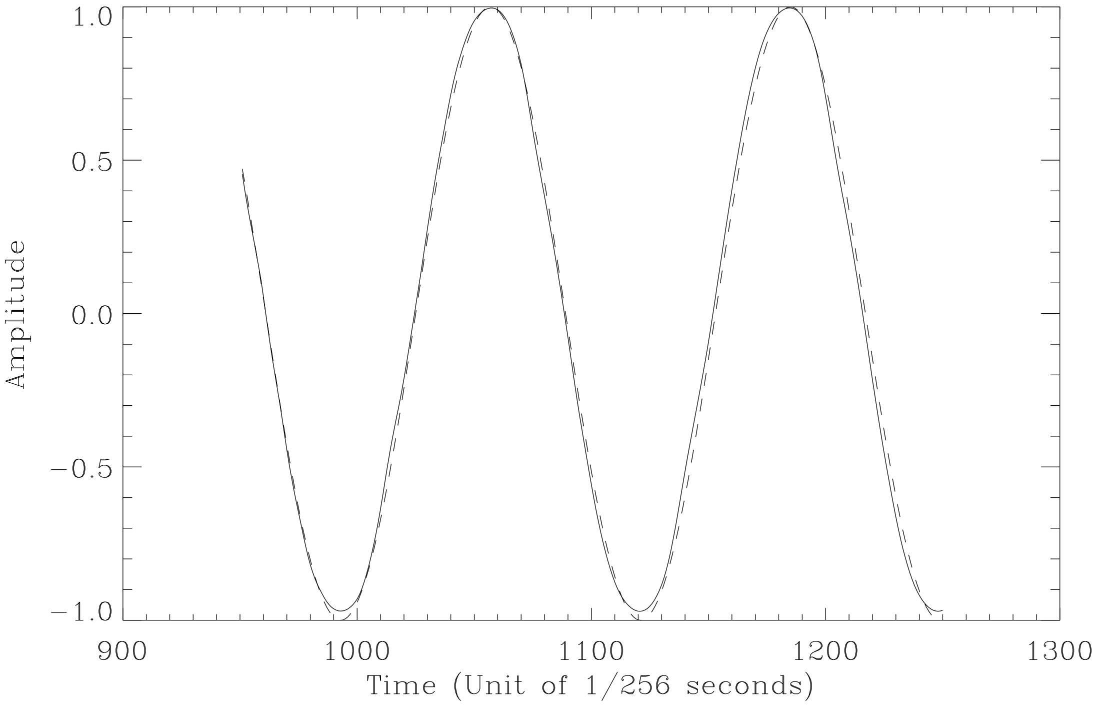
−1.0
−1.5
Amplitude
900
1000
1100
1200
1300
Time (unit of 1/256 seconds)

Figure 7(b)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−1.0
Amplitude
900
1000
1100
1200
1300
Time (Unit of 1/256 seconds)

Figure 8(a)

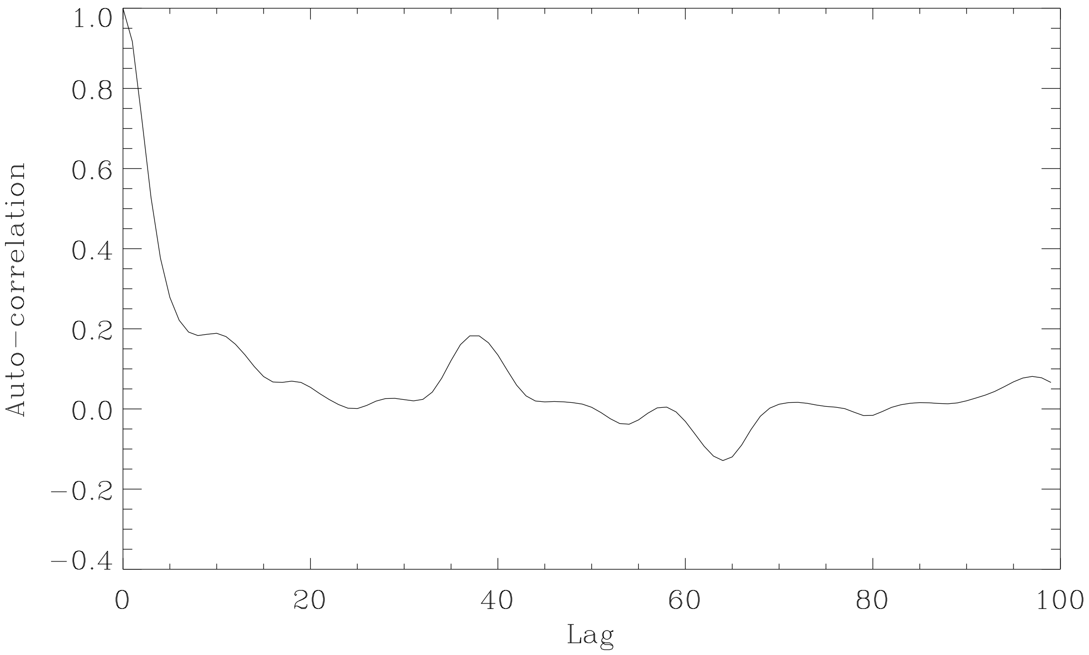


Figure 8(b)

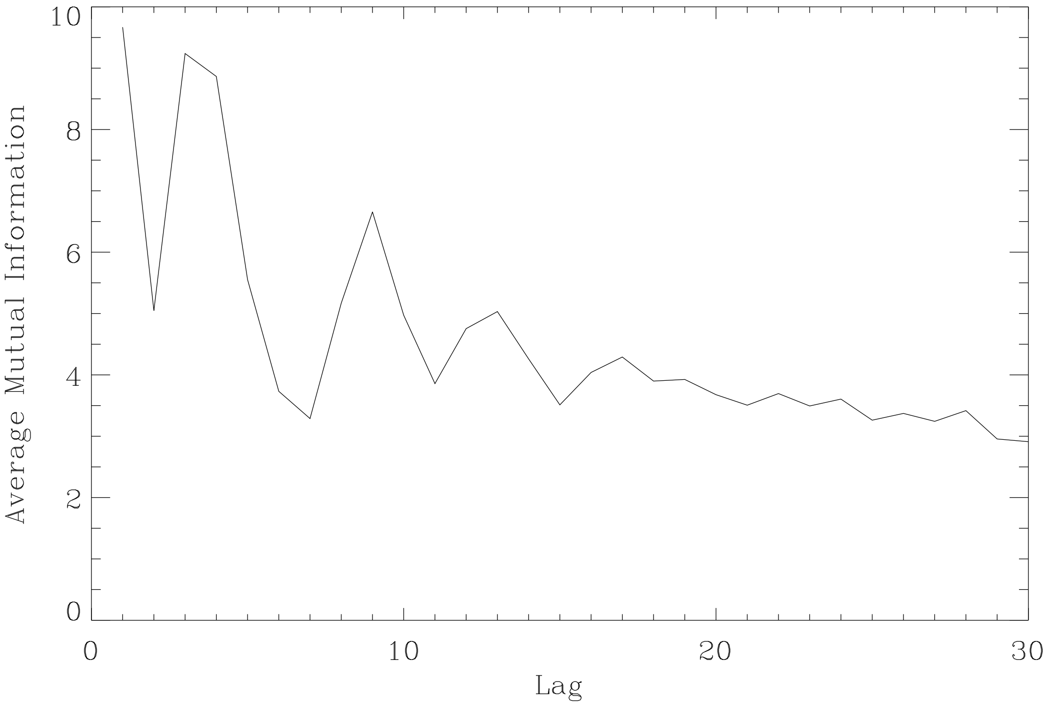


Figure 9

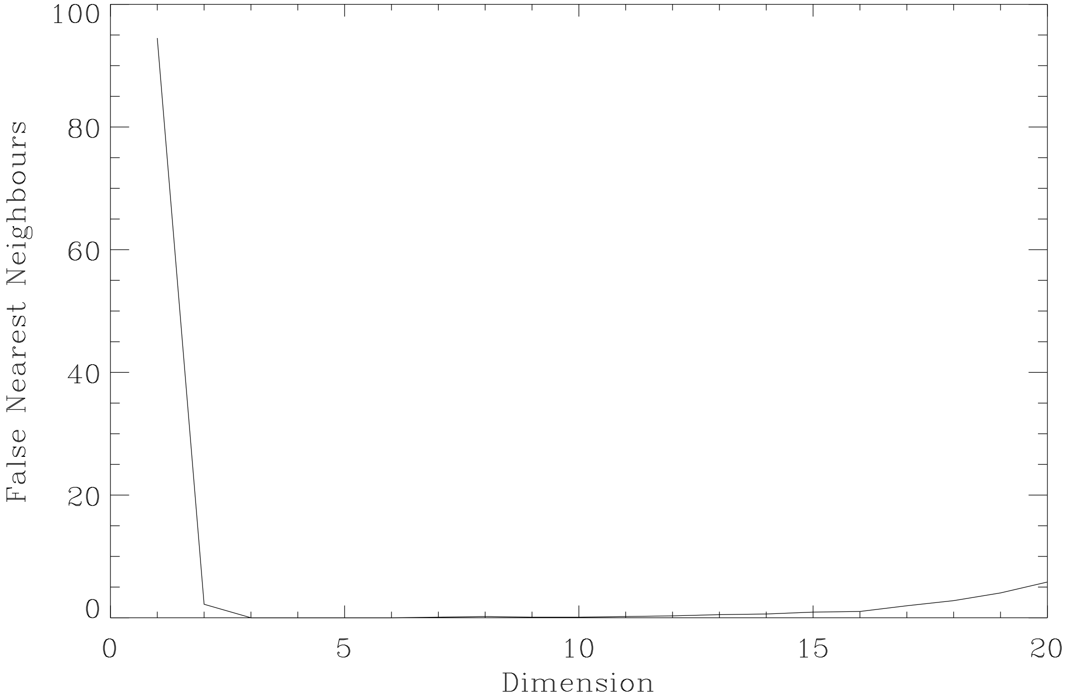

100
80
60
40
20
0
False Nearest Neighbours
0
5
10
15
20
Dimension

Figure 10

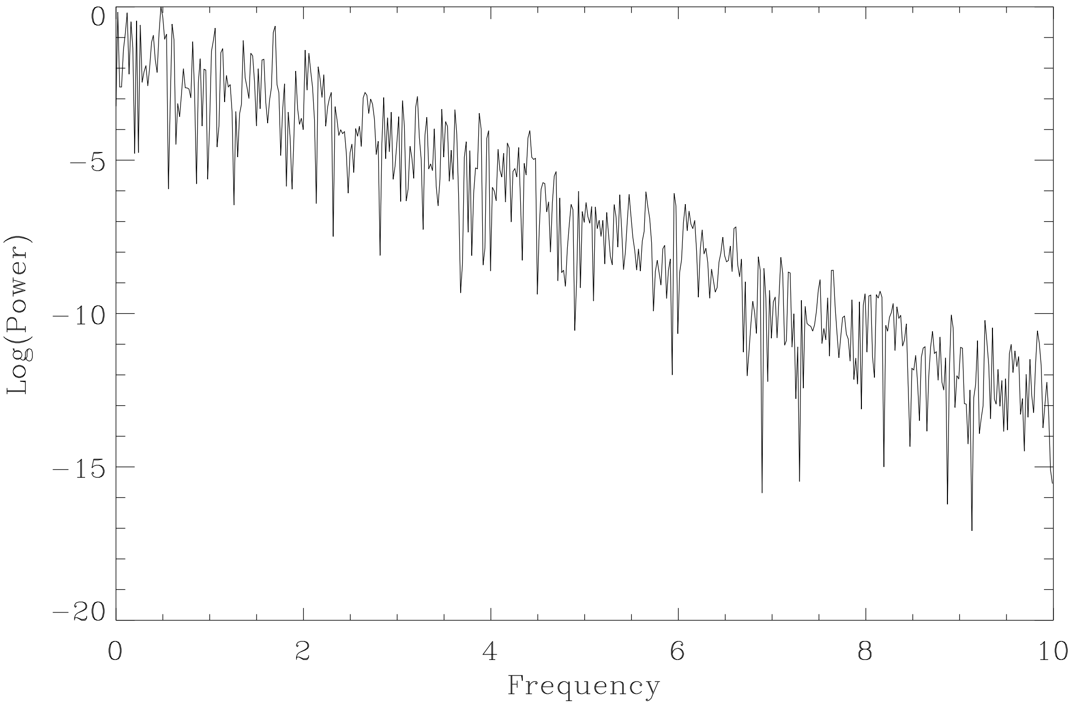


Figure 11

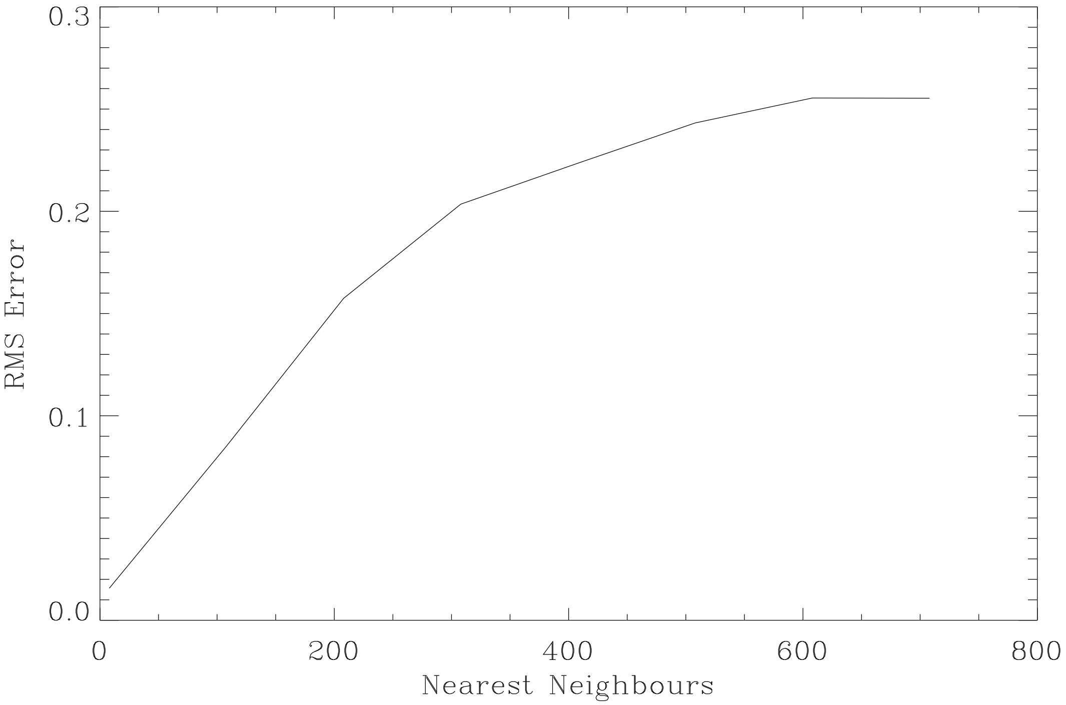


Figure 12

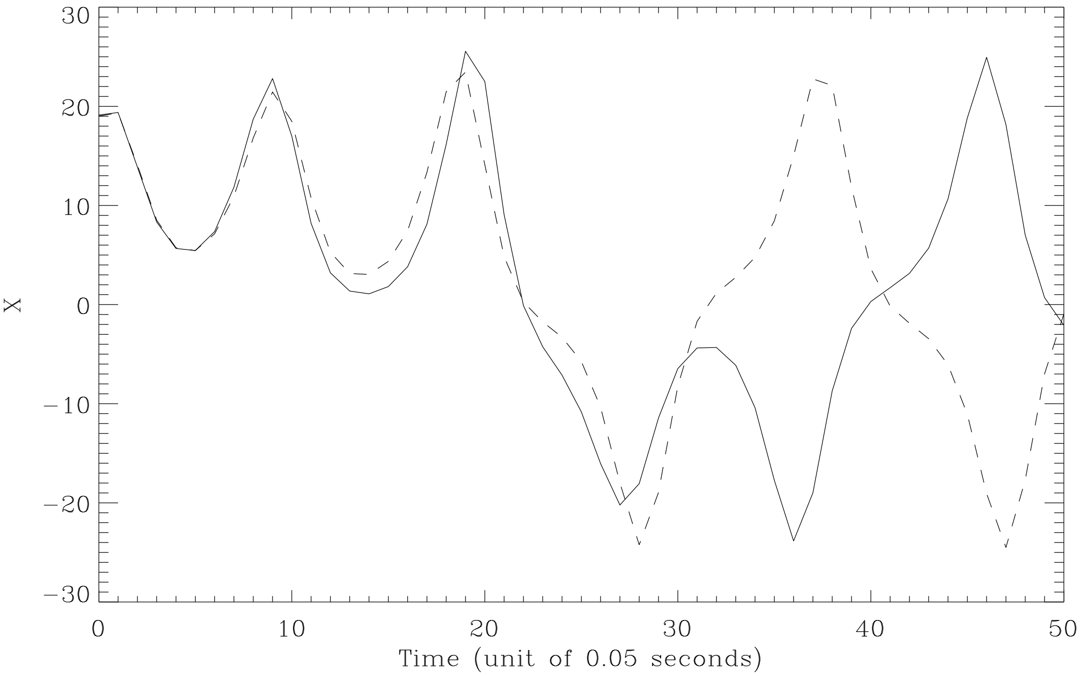


Figure 13

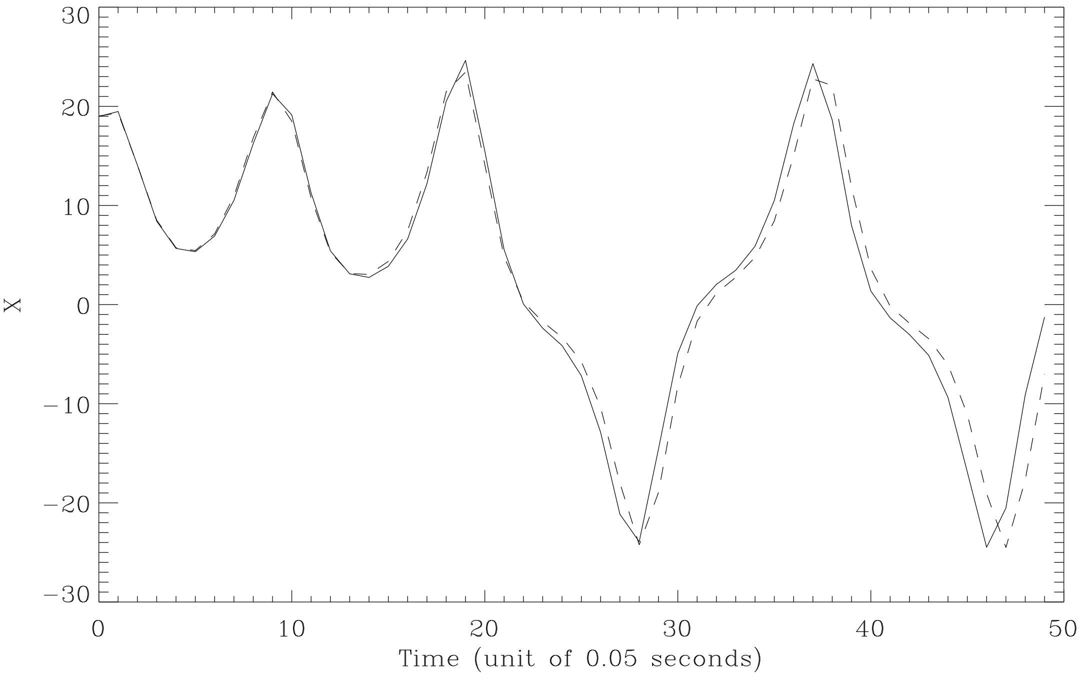


Figure 14

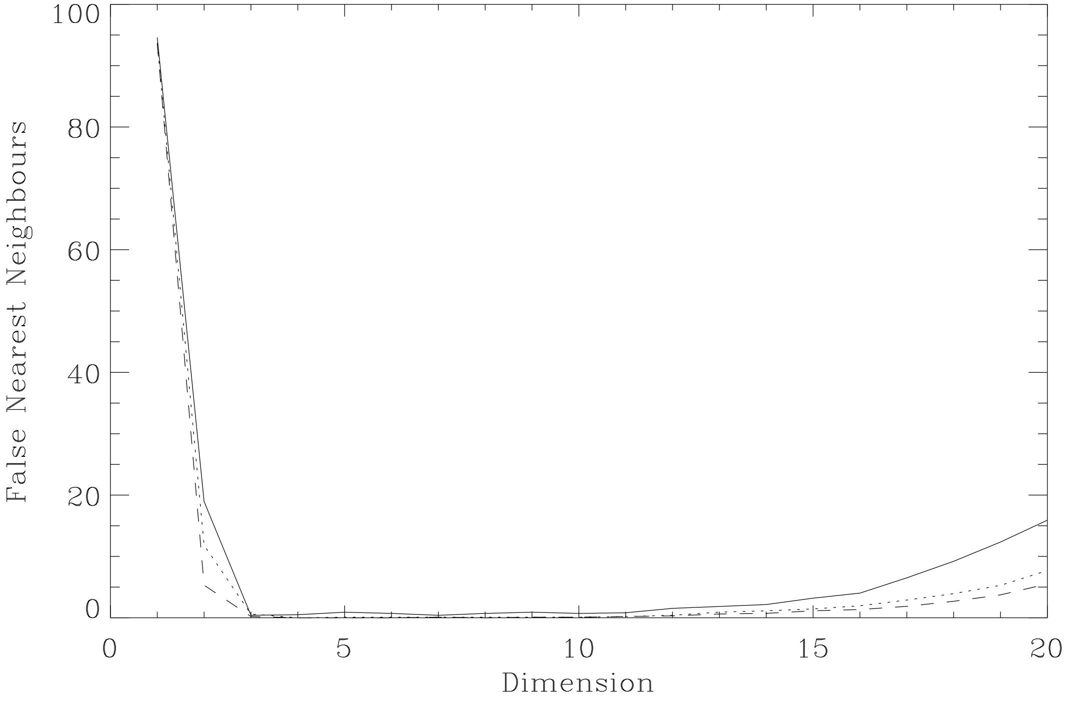
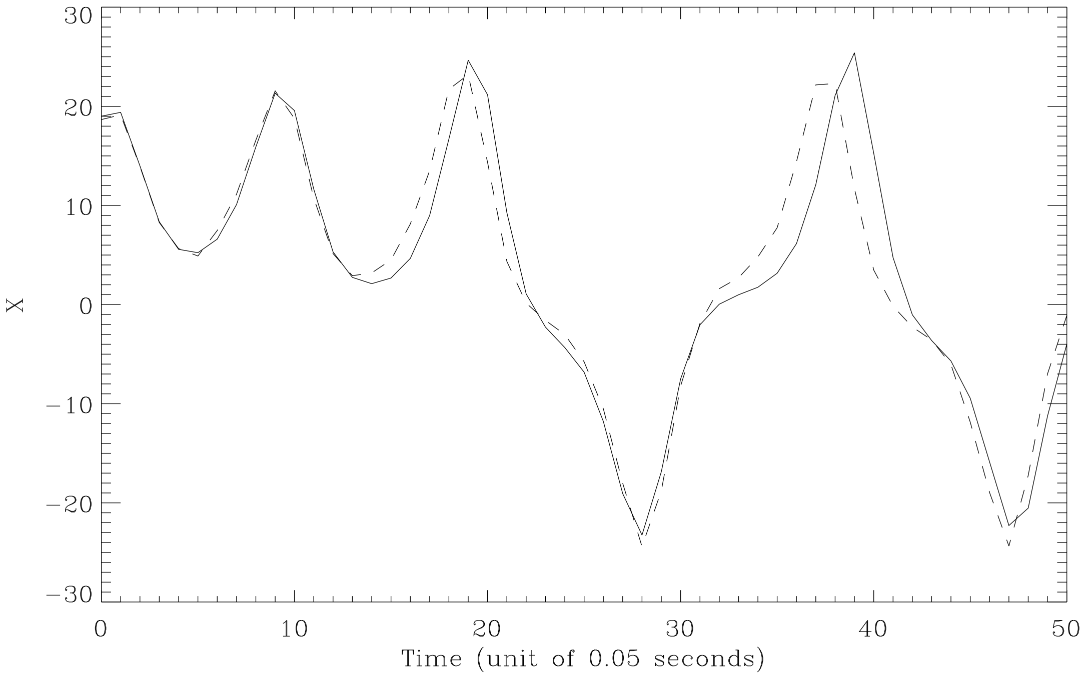

False Nearest Neighbours
100
80
60
40
20
0
0
5
10
15
20
Dimension

Figure 15


X
Time (unit of 0.05 seconds)